# Arizona Journal of Natural Philosophy

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PURPOSES OF THE JOURNAL:

1) To provide a forum for the free-thinking communication among all those interested in physics, mathematics, logic, philosophy, science and mathematics education, heuristics, artificial intelligence, and other closely related fields.

2) To publish essays, expository articles, and research articles on these subjects.

3) To promote the awareness of and further interest in the philosophical foundations of science and mathematics, and to promote the establishment of a modern inquiry of Natural Philosophy.

4) To investigate and define the purpose, scope, and methodology of modern science.

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Editor and Publisher: Patrick Reany
Mailing address: 2408 W Myrtle #26, Phoenix, AZ 85021
Phone: (602) 995-1637

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Arizona Journal Of Natural Philosophy

# ESSAYS:



# ARTICLES:

# Essays:

## Heuristics, the art of problem solving

by Patrick Reany

There is perhaps no more important area of study to the mathematician and physicist than that of *heuristics*, the theory and practice of problem solving. I can only guess why this subject is totally neglected in formal education, and I will try. Heuristics is an art form—a highly technical art form—but an art form just the same, and as such, the traditional rationalist is not likely to see the merit of teaching it. Western education is locked into a breakneck pace of force feeding students with rationalized learining that can't efficiently be retained and assimilated by the student, even by the time he or she has a doctorate. Thus we have the adage that a student doesn't really begin to comprehend the material until he or she teaches it. In fact, I do not fault this as a genuine technique, for if a student is faced with the *problem* of learning a subject well, it is wise to teach the subject to learn it.

But what I'm trying to fault is the outdated notion that Western education can continue to force feed students at this breakneck pace. I am reminded of the Eastern meditiation techniques where the student is taught in slow and deliberate procedure to incorporate subjective views into his own philosophy of life. Translated into Western methods of teaching, this approach would "slow down" the rate of "educating" the student, but would also produce a far better graduate. Traditional education in physics, for example, considers the student well educated if he or she does well in theory and experiment, but there is much more to contend with to be truly up-to-date with a robust education. Also needed is a foundation in the history and philosophy of physics, computer methods, ever more mathematical methods, and most importantly, heuristics.

I believe that the greatest offence to Rationality committed by Western education is its doctrinal myth that science and mathematics are devoid of subjectivity. This doctrine causes a generation gap between student and teacher. The student invariably sees problem solving as subjective, while the teacher is expected to present it as

objective and encompassed by neat sets of rules. The truth, however, is somewhere in between.

Do mathematicians and scientists conceal their heuristic techniques the same way that industries conceal their trade secrets? Perhaps some do. It's understandable, because in heuristics is the power not only to solve textbook problems, but also the power to be truly creative: to produce some new and wonderful result or theory that could make a reputation for someone. That's magic, magic worth keeping a secret. But that kind of magic belongs to us all.

# Articles:

## More proofs in plane geometry using vector methods

by Patrick Reany

In the last issue I promised to present two more geometry problems solved by the method of vectors, so here I go with them.

Problem 1). Show that in a right triangle the midpoint of the hypotenues is equidistant from the vertices.

We begin by drawing a figure such as Figure 1. Since the parts of the figure are already simple polygons we won't have to subtract anything until we subtract the regions, translating their information into circuit equations as is done in Table 1.

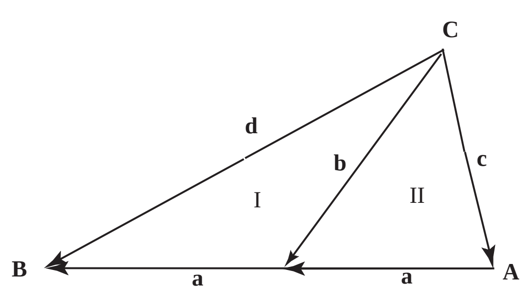



Figure 1

**Table 1** Circuit and structure equations from Figure 1.1.

| Circuit Equations | Structure Equations |
|---|---|
| I: $\mathbf{d} = \mathbf{b} + \mathbf{a}$<br>II: $\mathbf{c} = \mathbf{b} - \mathbf{a}$ | $\mathbf{d} \cdot \mathbf{c} = 0$ |

I need to show that $a = b$, where $a \equiv |\mathbf{a}|$, etc. To do this I will use the standard trick of substituting the circuit equations into the structure equation. Thus I get that

$$0 = \mathbf{d} \cdot \mathbf{c} = (\mathbf{b} + \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a}) = |\mathbf{b}|^2 - |\mathbf{a}|^2 = b^2 - a^2$$

Thus $b = a$, as I needed to show.

Problem 2) Show that the perpendicular bisectors of a triangle are concurrent at a point.

If you don't remember how you solved this problem in highschool, you might not see how to get started. The trick is to recognized that two of the perpendicular bisectors will meet at a point; then we must show that the vector from that intersection point to the midpoint of the third side is perpendicular to that third side. Thus we draw a figure as in Figure 2. From that figure we obtain the following table:

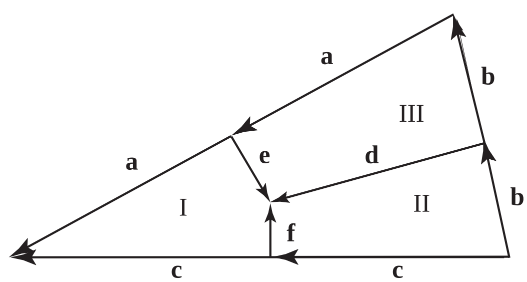



Figure 2

**Table 2** Circuit and structure equations from Figure 2.

| Circuit Equations | Structure Equations |
|---|---|
| I: $\mathbf{f} = \mathbf{c} - \mathbf{a} + \mathbf{e}$<br>II: $\mathbf{f} = -\mathbf{c} + \mathbf{b} + \mathbf{d}$<br>III: $0 = \mathbf{b} + \mathbf{a} + \mathbf{e} - \mathbf{d}$ | 1) $\mathbf{a} \cdot \mathbf{e} = 0$<br>2) $\mathbf{b} \cdot \mathbf{d} = 0$ |

We wish to show that $\mathbf{c} \cdot \mathbf{f} = 0$, which I will refer to as the *ShowThat equation*. The method from here is clear: we will substitute into the LHS of the ShowThat equation the circuit equations to derive the RHS of the ShowThat equation. The problem for us is to determine the best choice for $\mathbf{c}$ and $\mathbf{f}$ in terms of $\mathbf{a} \cdot \mathbf{e}$ and $\mathbf{b} \cdot \mathbf{d}$. We can use the following simplyfied expressions for $\mathbf{f}$ and $\mathbf{c}$:

I+II+III: $2\mathbf{f} = 2\mathbf{b} + 2\mathbf{e} \Rightarrow \mathbf{f} = \mathbf{b} + \mathbf{e}$

I−II−III: $0 = 2\mathbf{c} - 2\mathbf{b} - 2\mathbf{a} \Rightarrow \mathbf{c} = \mathbf{a} + \mathbf{b}$

We can now substitute these expressions into the LHS of the ShowThat equation:

$$\mathbf{c} \cdot \mathbf{f} = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} + \mathbf{e}) = \mathbf{b} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{e}) = \mathbf{b} \cdot \mathbf{d} = 0$$

As a bonus we can show that the concurrence point is equidistant from the three vertices, or that

$$|\mathbf{c} + \mathbf{f}|^2 = |\mathbf{c} - \mathbf{f}|^2 = |\mathbf{b} - \mathbf{d}|^2$$

It's easy to show that $|\mathbf{c}+\mathbf{f}|^2 = |\mathbf{c}-\mathbf{f}|^2$ because $\mathbf{c}\cdot\mathbf{f} = 0$. Using line II of the Table, it's easy to show that $|\mathbf{c} + \mathbf{f}|^2 = |\mathbf{b} - \mathbf{d}|^2$.

$$|\mathbf{c} + \mathbf{f}|^2 = |\mathbf{b} + \mathbf{d}|^2 = |\mathbf{b} - \mathbf{d}|^2$$

where the last step results because $\mathbf{b} \cdot \mathbf{d} = 0$.

# Heuristics 101

by Patrick Reany

This is the first of a series of articles on heuristics. This paper will focus on those general problem-solving techniques of widest scope, and in future articles I'll present more specific heuristics in mathematics and physics.

Heuristics can be divided into two groups of techniques: those that produce algorithms and those that don't. Heuristics can also be divided into those techniques that produce known solutions—the usual case with textbook problems—and those that produce something truly creative, unknown to anyone else.

The first step in preparing to solve a problem is to believe that you can solve it. The second step is to recognize the task you must perform as a formal problem susceptible to formal problem-solving techniques. I offer the following rough definition of a problem. A *problem* is a task that you cannot immediately conceive of an algorithm to solve it. An *algorithm* is a method of performing a task by dividing the task into clear, unambiguous steps whose execution is completable in a finite amount of time. A recipe is a good example of an algorithm.

One last point before beginning. Every time anyone arrives at a solution to a problem that is new to him or her, then a certain amount of intuition has been used. Intuition is that part of us that allows us to believe in things that we can't prove. Now I list my favorite heuristics, given in mostly random order. The exceptions are the first few, which I consider the most important. The first on the list I call *The Zeroth Rule of Problem Solving.*

0) Make any assumption necessary to solve the problem in a reasonable amount of time with a reasonable amount of effort.

1) "Keep an open mind—that's the secret." (Quote from *Doctor Who.*)

2) Be prepared to be a maverick. Everyone else may be doing something inefficiently or even incorrectly.

3) Even if you're a nobody, Nature doesn't mind: It reveals its secrets to nobodies who care. (Paraphrase from "The Galaxy Being," *The Outer Limits.*)

4) Neatness counts. Get organized and stay that way.

5) Invent your own algorithms and heuristics. Don't assume that those of your teacher, text, office or technical manual are best.

6) If the solution you obtain seems correct but doesn't jive with reality, check it again, redo it from the foundation up (the approach you used may contain false assumptions), question every piece of data (even a reference book can be in error).

7) If you're stuck in the middle of a solution, explain your solution to someone. Changing your relation to the problem from solver to explainer could give you an insight.

8) Be prepared to analyze, to break things down into their parts and to declare the relations between those parts. Remember, however, that there is no unique division of something into its "parts."

9) Be prepared to scrap conventional definitions, assumptions, and assignment of "parts."

10) If the problem you're solving is written out, be sure to interpret the meaning of the words in context.

11) Don't be afraid to ask yourself questions.

12) Determine the minimum number of things left for you to know to finish solving the problem.

13) Try to solve a simpler related problem.

14) Try to solve a more general problem by dropping one or more of the constraints of the problem.

15) If you can, divide a large problem into tractable subproblems.

16) Put the problem aside for a day or two.

17) Brainstorm for ideas, even wild ones. Determine which approaches will definitely *not* work.

18) Don't overlook the obvious.

19) Ask yourself what the next best thing is.

20) Guess.

21) If you know the answer but not the proof, work backward from the answer.

22) Look for metaphors from other fields, such as when Rutherford likened the atom to the solar system.

23) Study a variety of fields.

24) Try to turn a disadvantage into an advantage.

25) If your intuition tells you that there's got to be a better way to do something, there probably is.

26) Avoid limiting the applicability of an object as suggested by its commonly accepted name.

27) If you know all (or at least some) of the possible solutions, you can list each one (perhaps in a table) then test them one by one.

The above list is not exhaustive. Perhaps you can appreciate the discursive nature of the field as represented by the above heuristics, which range from the psychology of problem solving to the twilight of human creativity.

I now want to give some steps to becoming a better problem solver.

a) Think of yourself as a problem solver. After succeeding in solving a specific problem, intuit the general heuristics you used subconsciously. To do that you may have to carefully analyze what you have done.

b) Many good problem-solving techniques are used implicitly or explicitly in TV programs and novels. Look for them. There is a similarity between being a good problem solver and being a good detective.

c) Study logic and epistemology (the formal theory of knowledge). You didn't think it was going to be all TV and mystery novels, did you? If you want to progress beyond cranking formulas you have to study some philosophy.

d) Solve logic games and puzzles.

e) Study thoroughly the problem you need to solve. Be prepared to do research on others' attempts and approaches.

f) Always look for a better way.

g) Increase your vocabulary.

h) Read what other have written on heuristics.

i) Have a large number of problems-to-be-solved cooking on the back burners of your mind.

j) Don't neglect to master the fundamentals. The shovel that digs dirt can also dig gold.

k) Always ask yourself if the principle you've just learned is a special case of a more general principle.

l) Always reverse metaphors. For example, if atoms are like stars, in what sense can stars be likened to atoms. (Warning: If you tend to judge this problem as trivial and uninteresting, you haven't thought long enough on it. Sometimes the difference between those who make interesting discoveries and those that don't is merely a matter of attitude and perseverence.)

m) Categorize. Strive for *concinnity*.

# A neat "physics" problem

by Patrick Reany

This problem is from *University Physics* (Sears & Zemansky, Addison Wesley 4th ed), page 421: 29-10. It asks the student to prove that the resistance of the

following network is equal to $(1+\sqrt{3})r$.

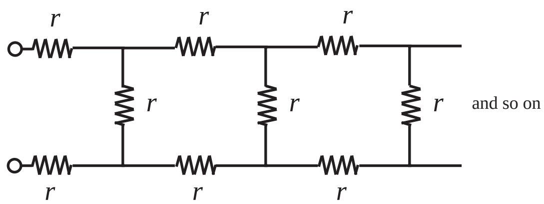



Figure 1

The very first fact that hit me when I attempted this problem is that it's not a true physics problem. It isn't at least if we define a "true physics problem" as one having an analog solution. This one doesn't because it has an unrealistic infinite series of loops. I decided right off that the heuristics needed to solve this problem are well beyond those of the average freshman physics students.

The first step in solving this problem is to solve a simpler related problem. Let's start with the one in Figure 2. Then I'll redraw it as is shown in Figure 3.

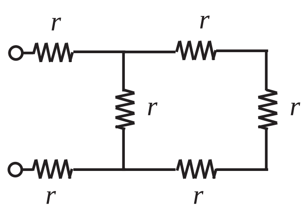



Figure 2

The resistance of the resistance combination in Figure 3 is $2r + \dfrac{r(3r)}{r+3r}$.

By adding another loop as in Figure 4, we might hope to find a pattern.

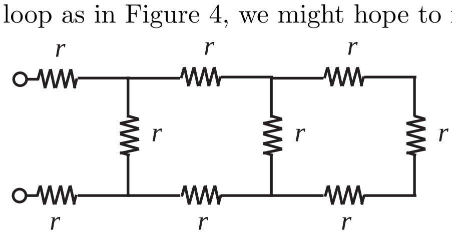
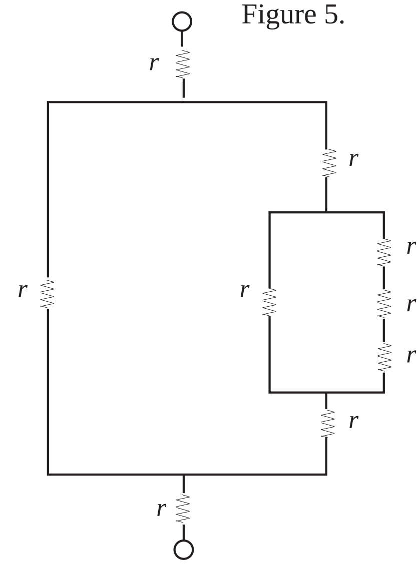



Figure 4

Figure 5.

On redrawing the figure we get Figure 5:




Now we can look for some pattern that is operating. Comparing Figure 5 with Figure 3, we can see that by adding a loop we have replaced the outlined part of Figure 3 with a copy of all of Figure 3.

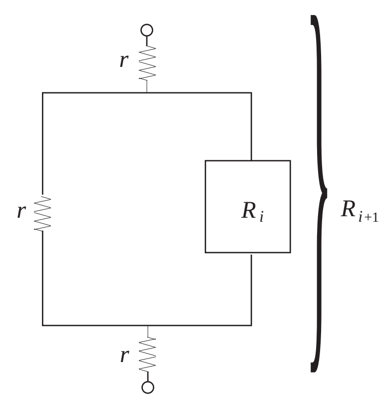



Figure 6

So now we have a recursive definition of the resistance. The resistance of the setup with $i+1$ loops, $R_{i+1}$, is a function of the resistance, $R_i$, of the setup with $i$ loops

$$R_{i+1} = 2r + \frac{rR_i}{r+R_i}$$

In the limit $i \to \infty$, $R_{i+1} = R_i \equiv R$ therefore,

$$R = 2r + \frac{rR}{r + R}$$

which is a quadratic with roots $(1 \pm \sqrt{3})r$.

Thus the "physical" root is $(1 + \sqrt{3})r$. In fact, this root cannot literally be interpreted as "physical."

This problem illustrates so many aspects of heuristics that I felt presenting it would help the reader see the partly pro forma, partly trial-and-error methodolgy of problem solving. My first few attempts to solve this problem I tried to find a recursive formula by thinking of each new circuit as being added onto the right side of the previous $n$-loop circuit. This instinctive method doesn't work, however. It turned out, upon much reflection, that the only way to proceed was to think of each new loop being added to the left of the previous $n$-loop system.

Perhaps I could have solved this problem *pro forma* if I had thought to examine all symmetries of the problem before proceeding further. In this case the symmetry is a "left-right" symmetry of the circuit diagram itself.